HYPER VERSION

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Complete solution for M(atrix) theory at two loops

Katrin Becker and Melanie Becker

California Institute of Technology 452-48, Pasadena, CA 91125
E-mail: beckerk@theory.caltech.edu, mbecker@theory.caltech.ed

Abstract: The complete result for the effective potential for two graviton exchange at two loops in M(atrix) theory can be expressed in terms of a generalized hypergeometric function.

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It has recently become clear that DLCQ supergravity and M(atrix) theory for finite N [1] are in perfect agreement even at the level of three graviton exchange [2].[1]

At this point all the calculations done in M(atrix) theory at two loops are based on a perturbative expansion of some complicated integrals and no closed expression for the effective action is known. Now more than ever before it will be important to have a systematic approach to calculations in M(atrix) theory. In this way it might be possible to go beyond computations in general relativity.

In this note we will make one step in this direction. We will present the complete result for the effective potential for two graviton exchange in M(atrix) theory at two loops. Our notation and conventions are those of [6].

The integrals that have to be solved for all diagrams appearing in double graviton exchange at two loops in M(atrix) theory are (after the appropriate substitution) of the type

$$I = \int_1^\infty dx dy \frac{(-1+x)^\alpha (-1+y)^\beta}{(-1+xy)^\gamma} x^{-\zeta+\mu} y^{-\zeta+\nu}. \tag{1}$$

Here $\zeta = b^2/2v$ and the other free parameters are constants whose value depend on the concrete integral. This expression can be evaluated exactly and its solution is

$$\begin{aligned} &B\,(\alpha+1, \beta+1)\, B\,(\zeta-\nu-\beta-1+\gamma, 2+\alpha+\beta-\gamma) \times \\ &\times {}_3F_2\,(1+\alpha, 2+\alpha+\beta-\gamma, 1+\alpha+\mu-\nu; 2+\alpha+\beta, \zeta+\alpha-\nu+1)\,, \end{aligned} \tag{2}$$

where $B(x,y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$ is the beta function and ${}_3F_2$ is a generalized hypergeometric series of unit argument. Using the Stirling series

$$\log\Gamma(z) = (z-1/2)\log z - z + 1/2\log(2\pi) + \sum_{n=1}^{m} \frac{B_{2n}}{2n(2n-1)} z^{2n-1} + O(z^{-2m-1})\,, \tag{3}$$

as z becomes large and the asymptotic expansion

$${}_3F_2(a,b,c;d,e) = 1 + \frac{abc}{de} + \ldots + \frac{(a)_n (b)_n (c)_n}{(d)_n (e)_n} + O(e^{-n-1})\,, \tag{4}$$

as e becomes large (while a, b, c and d are fixed numbers), we can obtain the double expansion of the effective potential to all orders in the velocity and the impact parameter as $\zeta \to \infty$. Here B_{2n} are the Bernoulli numbers whose numerical values and recurrence relations can be found in Ramanujan [7].

The complete solution for the effective potential coming from the fermions is

$$\begin{aligned} \mathcal{F} = &-\frac{4\sqrt{2}\pi}{v^{3/2}} \frac{\Gamma(\zeta+1)}{\Gamma(\zeta+\frac{3}{2})} {}_3F_2(\frac{1}{2},\frac{3}{2},\frac{3}{2};3,\zeta+\frac{3}{2}) - \\ &- \frac{135\sqrt{2}\pi}{512 v^{3/2}} \frac{\Gamma(\zeta+1)}{\Gamma(\zeta+\frac{7}{2})} {}_3F_2(\frac{5}{2},\frac{5}{2},\frac{7}{2};5,\zeta+\frac{7}{2}) + \frac{32\sqrt{2}\pi}{v^{3/2}} \frac{(-\Gamma(\frac{1}{2}+\zeta)^2 + \Gamma(\zeta)\Gamma(1+\zeta))}{\Gamma(\zeta)\Gamma(\frac{1}{2}+\zeta)}\,. \end{aligned} \tag{5}$$

[1]Previous work indicated that such an agreement might be possible [3, 4, 5].

Incidentally, one of the hypergeometric series could be summed up using Dougall's theorem. The complete solution coming from bosons and ghosts is given by the odd terms in v of the formula

$$\mathcal{B} = \frac{\pi\sqrt{2}}{3v^{3/2}}\frac{\Gamma(-\frac{1}{2}+\zeta)}{\Gamma(\zeta)}\Big[\frac{49}{8}{}_3F_2(\frac{1}{2},\frac{1}{2},\frac{1}{2};1,\zeta) - {}_3F_2(\frac{3}{2},\frac{3}{2},\frac{3}{2};3,1+\zeta) - \\ - \frac{137}{16}\frac{(2\zeta-1)}{\zeta}{}_3F_2(\frac{1}{2},\frac{1}{2},\frac{1}{2};1,1+\zeta)\Big]\,. \tag{6}$$

If we expand these formulas using the Stirling series we obtain the effective potential computed in [6] up to order v^8. The leading order corresponds to the relativistic correction of the v^4-term of [8], while the higher order terms correspond to quantum gravity corrections.

The complete result of the effective potential of M(atrix) theory at one loop was found by DKPS [9]. It sounds plausible that the complete solution at one and two loops may teach us how to solve the model for an arbitrary number of loops.

Acknowledgments

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